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PROGRAM MANUAL FOR MINIMUM VARIANCE PRECISION TRACKING AND ORBIT PREDICTION PROGRAM

BY

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GREENBELT, MD.

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for

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I. INTRODUCTION

This report describes a prototype version of a computing program for orbit determination based on an extension of the Schmidt - Kalman minimum variance technique for processing tracking data.

The analysis on which the program is based was carried out by Samuel Pines and Henry Wolf of Analytical Mechanics Associates, Inc. with assistance from R. K. Squires and D. S. Woolston of the Goddard Space Flight Center, NASA, and from Mrs. A. Bailie of AMA. The digital program for the IBM 7090 or 7094 was written by John Mohan of AMA. The program makes use of portions of a least squares orbit determination program written by Miss E. Fisher of Goddard Space Flight Center. Significant contributions to the program have been made by J. Behuncik and G. Wyatt of Goddard Space Flight Center who have also assisted in checking it out.

II. NOTATION

A, E	Azimuth and elevation, respectively
$\frac{A\,C_D}{m}$	Drag parameter for the vehicle; A is effective frontal area in cm^2, m is the mass in grams, C_D is the drag coefficient
a	Semi-major axis
D	Declination angle
d_r	Vector dot product, $R_r \cdot \dot{R}_r$
$E(x)$	Expected value of the variable x
F_1, F_2, F_3	Peturbation acceleration vectors
f, g, f_t, g_t	Functions used in solving the two-body problems; see eq. (12)
f_1, f_2, f_3, f_4	Trigonometric functions defined by equations (13) and (14)
H	The angular momentum vector $H = R \times \dot{R}$ with components H_x, H_y, H_z
J_2, J_3, J_4	Oblateness coefficients of the earth
$L(t)$	Modified Kalman filter, equation (21)
ℓ, m, n	Minitrack direction cosines
$M(t)$	Matrix of partial derivatives of the observations with respect to the instantaneous state variables
$N(t)$	Matrix of partial derivatives of the observations with respect to the orbit parameters
n	Mean motion

$Q(t)$, $P(t)$, $Y(t)$	Covariance matrices of the orbit parameters, the state variables, and of the estimated errors in the observations, respectively
$R = R_{vc}$	Position vector of vehicle relative to the dominant body
$\dot{R}$	Velocity vector relative to the dominant reference body
$\ddot{R}$	Acceleration vector relative to the dominent reference body
R_{ci}	Position vector of the i^{th} mass with respect to the reference body
R_i	Vector position of the vehicle from the i^{th} mass
R_r, $\dot{R}_r$	Initial position and velocity vectors of the osculating two-body orbit
R_{TB}	Position vector of the local two-body orbit used in the Encke method
R A	Right ascension
r	Scalar value of the vector R
r_r	Scalar value of R_r
r_{TB}	Scalar value of the vector R_{TB}
$S(t)$	Point transformation matrix of the partial derivatives of the coordinates with respect to the parameters
t	Time
t_o	Time of an observation
t_r	Time of rectification

t^+, t^-	The instant of time immediately following t and preceding t respectively
v	Scalar value of the vector $\dot{R}$
x, y, z	Rectangular cartesian coordinates of the vector R
x_s, y_s, z_s	Inertial geocentric coordinates of the tracking station
x''', y''', z'''	Topocentric coordinates of the vehicle in the local horizon, local vertical system with x''' positive south, y''' positive east, and z''' positive upward along the local vertical, see equation (40)
$\dot{x}$	Time derivative of the vector x
α	Orbit parameter
$\alpha_i (t)$	The i^{th} parameter evaluated at time t
$\Delta \alpha (t)$	Deviation in the orbit parameters
$\Delta x (t)$	Deviations in the state variables
$\overline{\epsilon^2}$	Covariance matrix of the observation instrument errors
θ	Incremental eccentric anomaly
θ'	Right ascension of the tracking station meridian
μ	Product of the universal gravitational constant and the mass of the dominant reference body
ξ	Encke perturbation displacement vector
ρ	Mass density of the atmosphere as used in equation (8)
ρ	Scalar range distance of the vehicle from a tracking station as used in equation (33) et seq.

$\dot{\rho}$	Range rate
$\Phi (t, t_o)$	Transition matrix of the state variables
φ	Geodetic latitude
$\psi (t, t_o)$	Approximating form of the parameter transition matrix
Ω	Angular velocity vector of the dominant reference body as used in equation (8); also right ascension of the ascending node as used in discussing the osculating elements
$\Omega (t, t_o)$	Transition matrix of the variational parameters
ω_e	z component of the earth's siderial rotation rate
α_1	A rigid rotation about the initial velocity vector
α_2	A rigid rotation about the initial position vector
α_3	A rigid rotation about the initial angular momentum vector
α_4	A change in the variable, $\frac{R \cdot \dot{R}}{\sqrt{\mu \mid a \mid}}$
α_5	A change in $\frac{1}{a}$
α_6	A change in $1 - \frac{r}{a}$

Vector Operations

A^*	Transpose of the matrix A
A^{-1}	Inverse of the matrix A
A X B	Vector cross product of the vectors A and B

III. ANALYTICAL BACKGROUND

A detailed discussion and derivation of the Schmidt-Kalman minimum variance technique as modified and extended for use in the present program is contained in reference 1. In the present document emphasis will be placed on the working equations required to implement the method. Some essential concepts and distinctive features of the method should be introduced, however, and are described briefly in this section.

A. Orbit prediction program

For the purpose of predicting position as a function of time the program makes use of Encke's form of the differential equations of motion. It therefore solves analytically the best local two-body problem and numerically integrates the deviation from this two-body reference trajectory. When the true trajectory departs from the reference orbit by preset limits, a rectification is performed; that is, the current true radius and velocity vectors are used to define a new reference path.

B. Solution of the two-body problem

In the particular solution of the two-body problem used herein problems associated with circular orbits and orbits of very low inclination have been eliminated by expressing the solution in terms of initial position and velocity vectors rather than vectors based on position of perigee.

C. Orbit Parameters

The choice of the elements used in the differential correction scheme is of utmost importance in predicting observations and other orbit functions over a long time period. It may be shown (Reference 2) that the best choice for parameters is that in which only one variable affects the energy or the mean motion of the orbit. The conventional astronomical elements have this property. However, three of these variables, the argument of perigee, the time of perigee passage, and the ascending node become poorly defined for near circular and low inclination orbits. The initial position and velocity components do not have this difficulty. However, all six of these affect the energy. A convenient set of parameters have been derived in Ref. 3. These avoid the difficulties for circular and low inclination orbits as well as restricting the energy parameters to a single element. The variational parameters are as follows:

$\Delta\alpha_1$ (t) A rigid rotation of R about $\dot{R}$ such that $R \cdot \dot{R}$ remains constant.

$\Delta\alpha_2$ (t) A rigid rotation of $\dot{R}$ about R such that $R \cdot \dot{R}$ remains constant.

$\Delta\alpha_3$ (t) A rigid rotation of R and $\dot{R}$ about H.

$\Delta\alpha_4$ (t) A change in the variable $\frac{R \cdot \dot{R}}{\sqrt{\mu |a|}}$ such that the angle between R and $\dot{R}$ is changed leaving the magnitude of R and $\dot{R}$ and also a unchanged.

$\Delta\alpha_5$ (t) A changing $\frac{1}{a}$ the reciprocal of the semi-major axis, such that the magnitudes of R and $\dot{R}$ are changed, leaving the eccentricity unchanged.

$\Delta\alpha_6$ (t) A change in the variable $1-\frac{r}{a}$, changing the magnitude of R and $\dot{R}$ and the angle between them such that a and $R \cdot \dot{R}$ are not changed.

These six elements have the characteristic that they determine the orbit independently of its orientation or shape and do not break down. Moreover, the matrix of partial derivatives of these elements contains only one secular term, namely that due to the semi-major axis, a.

D. The Schmidt-Kalman Technique

Since the orbit position and velocity are not directly observable, it is necessary to infer these variables from a sequence of observations which are functions of the trajectory. In the conventional methods, a linear relationship is assumed between the deviations in the observations and the corresponding deviations in the orbit variables. Thus, an error in the orbit position will correspond to some predictable error in the observation. A large number of observations are made, overdetermining the linear system of equations. A least square technique is used to obtain the best value of the orbit errors to fit the known observation errors. Since the equations of motion are essentially nonlinear, this region of linearity becomes more and more constrictive about the nominal trajectory the longer the time period over which the prediction is made. Thus, the least square technique often produces a result, fitting data over a long time arc, which is outside the linear range. This produces problems in convergence and consumes machine time. Reducing the number of observations to a shorter time arc helps avoid this difficulty. The weighted least squares is often used in this manner. However, a large number of observations is always needed in order to properly evaluate the effect of the random instrument errors.

The method of least squares and weighted least squares both relate the estimate of the initial parameters to an entire sequence of observational residuals spread over an extended time arc. In contrast, the method of minimum variance relates the present estimate of the state variable deviation to the present actual deviations in the observations. The linear assumptions required for the updating theory are violated to a much less degree in the method for minimum variance than in the method of weighted least squares.

Closed form analytical derivatives

The requirement for utilizing closed form analytical derivatives is associated with the need for rapid computing times. If the program were required to integrate the variations in the observations due to changes in the orbit parameters directly from the differential equations for these variations, the computing time over and above that required for the nominal trajectory would increase by a factor of six (6). Since these variations are only required in order to obtain small iterative changes to the orbit parameters, approximate expressions will be useable, provided the residual in the observations can be accurately computed. This situation is analogous to the possible use of an approximate derivative in Newton's method for obtaining the roots of a polynomial. The program presented in this report uses a set of analytical derivatives based on the two-body problem approximation of the osculating orbit given in terms of the parameters outlined earlier. In a manner similar to the Encke method, a readjustment is made in the partial derivatives whenever the orbit is rectified.

(As indicated in Section VI, the program does have the capability of obtaining the required partial derivatives numerically by use of the secant method; that is, by integrating the nominal orbit and six variation orbits in each of which one of the initial conditions has been slightly altered.)

IV. ORBIT PREDICTION PROGRAM

A. Equations of Motion

The equations of motion of a vehicle with negligible mass under the action of a dominant central force field and perturbed by other smaller forces is given by:

$$\ddot{R}_{VC} = - \frac{\mu}{r^3} R_{VC} + F_1 + F_2 + F_3 . \qquad 1)$$

These equations may be written in the Encke form by replacing the vector R_{VC} by the sum of a local two-body orbit position vector, R_{TB}, plus a perturbation displacement,

$$R_{VC} = R_{TB} + \xi . \qquad 2)$$

The vector R_{TB} satisfies the differential equation,

$$\ddot{R}_{TB} = - \mu \frac{R_{TB}}{r_{TB}{}^3} \qquad 3)$$

The Encke equation of motion for the perturbation displacement, ξ, is given by:

$$\ddot{\xi} = - \mu \left(\frac{R_{VC}}{r_{VC}{}^3} - \frac{R_{TB}}{r_{TB}{}^3} \right) + F_1 + F_2 + F_3 . \qquad 4)$$

These are the equations that will be integrated to obtain a precision nominal trajectory.

The perturbations that are included in this program are those due to the gravitational attraction of the sun, moon, Venus, Mars, and Jupiter (F_1); the program also includes the perturbations due to the earth's oblateness (F_2) and the perturbations due to atmospheric drag (F_3).

B. Computation of perturbation terms

The equations for the gravitational perturbation acceleration due to the sun, moon, and planets are given by:

$$F_1 = \sum_{i=1}^{5} \mu_i \left(\frac{R_i}{r_i^3} - \frac{R_{ci}}{r_{ci}^3} \right) \tag{5}$$

The perturbation accelerations due to the earth's oblateness are given by:

$$F_2 = bR_{VC} + c\hat{k} \tag{6}$$

where $\hat{k}$ is a unit vector in the z-direction and where

$$b = \mu \left[\frac{J_2}{r^5} \left(-1 + 5 \left(\frac{z}{r}\right)^2 \right) + \frac{J_3}{r^6} \frac{5}{2} \left(3 \frac{z}{r} - 7 \frac{z^3}{r^3} \right) + 15 \frac{J_4}{r^7} \left(-1 + 14 \frac{z^2}{r^2} - 21 \frac{z^4}{r^4} \right) \right] \tag{7}$$

$$c = \mu z \left[-\frac{2J_2}{r^5} + \frac{J_3}{r^5 z} \left(-\frac{3}{2} + \frac{15}{2} \frac{z^2}{r^2} \right) + \frac{20J_4}{r^7} \left(-3 + 7 \frac{z^2}{r^2} \right) \right].$$

in which r is the magnitude of the vector R_{VC}

The perturbations due to atmospheric drag are given by:

$$F_3 = \cdot \frac{1}{2} \rho \frac{AC_D}{m} \left(\dot{R}_{VC} - \Omega \times R_{VC} \right) \left| \dot{R}_{VC} - \Omega \times R_{VC} \right| \qquad 8)$$

The vector $\dot{R}_{VC} - \Omega \times R_{VC}$ is the velocity of the vehicle relative to the atmosphere rotating rigidly with the earth. The vector Ω is the earth rotation vector and contains only a component in the z direction. Its magnitude is given by the earth's siderial rotation rate.

C. Computation of the Encke term

A special problem arises in the computation of the Encke term due to the loss of accuracy in subtracting the nearly equal terms involved. An expression based on the binomial expansion removes this difficulty. This method supplies results more accurate than the straight-forward computation for terms of the type of:

$$\frac{R}{r^3} - \frac{R_o}{r_o^{\,3}}$$

if $|R - R_o|$ is small compared to r and is known more accurately than can be computed by taking the difference between R and R_o.

One can write

$$\frac{R}{r^3} - \frac{R_o}{r_o^{\,3}} = \frac{R}{r^3} - \frac{R}{r_o^{\,3}} + \frac{R}{r_o^{\,3}} - \frac{R_o}{r_o^{\,3}}$$

$$= \frac{R}{r^3} \left[1 - \left(\frac{r}{r_o} \right)^3 \right] + \frac{\Delta R}{r_o^{\,3}}$$

$$= \frac{\Delta R}{r_o^{\,3}} + \frac{R}{r^3} \left[1 - (1 + u)^{3/2} \right]$$

or finally:

$$\frac{R}{r^3} - \frac{R_o}{r_o^{\,3}} = \frac{\Delta R}{r_o^{\,3}} + \frac{R}{r^3}\sum_{n=1}^{6} a_n\, u^n \qquad 9)$$

where

$$u = \frac{2}{r_o^{\,2}}\left(R_o + \frac{1}{2}\,\Delta R\right)\cdot \Delta R$$

$$a_1 = -\frac{3}{2}, \quad a_2 = -\frac{3}{8}, \quad a_3 = \frac{1}{16}, \quad a_4 = -\frac{3}{128}, \qquad 10)$$

$$a_5 = \frac{3}{256}, \quad a_6 = \frac{-7}{1024}$$

The six terms are adequate for $|u| < 0.1$. For larger values of u straight-forward computation is adequate

D. Solution of the Two-Body Problem

The vector position and velocity for a Kepler orbit may be written in terms of the initial position and velocity vectors as given in Reference 4.

$$R_{TB} = fR_r + g\dot{R}_r \qquad 11)$$

$$\dot{R}_{TB} = f_t R_r + g_t \dot{R}_r \ .$$

The functions f and g can be expressed for both elliptic and hyperbolic orbits by:

$$f = -\frac{|a|}{r_r} f_2 + 1$$

$$g = -\frac{1}{n} f_1 + (t - t_r) \qquad 12)$$

$$\frac{r_c}{|a|} = f_2 + \frac{r_r}{|a|} f_4 + \frac{d_r}{\sqrt{\mu \, |a|}} f_3 = F'(\theta)$$

$$f_t = -\sqrt{\frac{\mu}{|a|}} \frac{1}{r_r} \frac{|a|}{r_c} f_3$$

$$g_t = -\frac{|a|}{r_c} f_2 + 1$$

$$n(t - t_r) = f_1 + \frac{r_r}{|a|} f_3 + \frac{d_r}{\sqrt{\mu \, |a|}} f_2 = F(\theta)$$

The functions f_1, f_2, f_3, f_4 are defined in terms of the incremental eccentric anomaly $\theta = E - E_r$.

For the elliptic case

$$
\begin{aligned}
f_1(\theta) &= \theta - \sin\theta \\
f_2(\theta) &= 1 - \cos\theta \\
f_3(\theta) &= \sin\theta = \theta - f_1(\theta) \\
f_4(\theta) &= \cos\theta = 1 - f_2(\theta)
\end{aligned}
\tag{13}
$$

For the hyperbolic case

$$
\begin{aligned}
f_1(\theta) &= \sinh\theta - \theta \\
f_2(\theta) &= \cosh\theta - 1 \\
f_3(\theta) &= \sinh\theta = \theta + f_1(\theta) \\
f_4(\theta) &= \cosh\theta = 1 + f_2(\theta)
\end{aligned}
\tag{14}
$$

where

$$
\begin{aligned}
r_r &= (R_r \cdot R_r)^{1/2} \\
d_r &= R_r \cdot \dot{R}_r \\
v_r^2 &= \dot{R}_r \cdot \dot{R}_r \\
a &= \left(\frac{2}{r_r} - \frac{v_r^2}{\mu} \right)^{-1} \\
n^2 &= \frac{\mu}{|a|^3} \; .
\end{aligned}
\tag{15}
$$

E. Integration and Rectification Control

The Encke method reduces somewhat the relative advantages of one integration scheme over another insofar as numerical accuracy is concerned. The method is capable of using almost any integration scheme to obtain a precise solution. The major advantage to be gained in the choice of integration schemes lies in the choice of the maximum integration interval to minimize the total computing time required. The Encke method computes the solution of the equations of motion as a sum of the exact function plus the integrated effect of the perturbations. Thus the solution may be kept as precise as the exact portion so long as the accumulated error in the integrated portion is kept from affecting the least significant digit of the exact term. By estimating the accumulated round-off error and the accumulated truncation error, in the integrated portion of the solution, and by rectifying the solution to a new osculating Kepler orbit, whenever the integrated error threatens to affect the least significant digit of the exact solution, the total solution may be kept as precise as the exact term can be computed.

The particular program outlined in this report uses a fourth order Runge-Kutta integration scheme to initialize a sixth order backward difference second sum Cowell integration formula. A constant step size is used in place of a variable integration interval. At pre-set points in the trajectory the optimum interval size is altered, based on previous numerical experience with these intervals.

The rectification feature outlined above, based on round-off error control, is presently not in the program. At present, rectification is triggered whenever the integrated portion of the solution is a fixed ratio of the exact two-body term. In effect, this controls the accumulation of round-off error.

The rectification control for switching reference bodies is triggered as a function of the relative scalar distances to the various attracting bodies. The radius of the sphere of influence of each body is pre-set in the program. Whenever the vehicle enters the sphere of influence of each body, the program rectifies the orbit, and recomputes the planetary coordinates so that the distances and velocities of the various bodies are measured from the new dominant reference body.

V. THE MODIFIED SCHMIDT-KALMAN METHOD

A. The Statistical Filter

A modification of the Schmidt-Kalman equations in terms of the new orbit parameters selected for use in this program has been derived in references 1 and 3 and is available for incorporation in the orbit determination program.

The deviations of the orbit variables in terms of the new parameters are given by

$$\Delta x(t) = \left(\frac{\partial x}{\partial \alpha}\right) \Delta\alpha(t) = S(t)\,\Delta\alpha(t) \qquad 16)$$

where S(t) is a point transformation matrix.

The parameter transition matrix $\Omega(t,t_o)$ is defined by

$$\Delta\alpha(t) = \left(\frac{\partial\alpha(t)}{\partial\alpha(0)}\right) \Delta\alpha(0) = \Omega(t,t_o)\ \Delta\alpha(0). \qquad 17)$$

The observation errors in terms of the new parameters are given by

$$\Delta\gamma(t) = N(t)\ \Delta\alpha(t) \qquad 18)$$

where

$$N(t) = M(t)\ S(t)\ .$$

The corresponding covariance matrices are given by

$$E(\Delta\alpha,\ \Delta\alpha^*) = Q(t) = \Omega Q(0)\ \Omega^*$$

$$P(t) = SQ(t)\ S^* \qquad 19)$$

$$Y(t) = NQ(t)\ N^* + \sigma^2\ .$$

The inverse relationship between the orbit parameter corrections and the observational errors is given by

$$\Delta\alpha(t) = L(t)\ \Delta\gamma(t)\ . \qquad 20)$$

The optimum filter $L(t)$ is given by

$$L(t) = QN^*\ Y^{-1} \qquad 21)$$

The corrected covariance matrix after each observation is given by

$$Q(t^+) = Q(t^-) - Q(t^-)\ N^*\ Y^{-1}\ NQ\ (t^-)\ . \qquad 22)$$

Using these equations, it is now possible to use the Schmidt-Kalman scheme for both short and long term predictions.

B. The Point Transformation Matrix

The inverse point transformation matrix, $S^{-1} = \frac{\partial \alpha}{\partial x}$, is given in reference 1 as

$$S^{-1} = \begin{bmatrix} -\frac{v}{h^2} H & 0 \\ 0 & \frac{r}{h^2} H \\ 0 & \frac{H \times \dot{R}}{hv^2} \\ \frac{H \times R}{a^2 n r^2} - \frac{a}{r^3} \alpha_4 \alpha_6 R & -\frac{H \times \dot{R}}{a^2 n v^2} - \frac{2a}{rv^2} \alpha_4 \alpha_6 \dot{R} \\ -\frac{2R}{r^3} & -\frac{2}{\mu} \dot{R} \\ \frac{v^2}{\mu r} R & \frac{2r}{\mu} \dot{R} \end{bmatrix} \tag{23}$$

By choosing $\frac{1}{a}$ as a parameter, the other five parameters will automatically be independent of the energy providing the inverse of the matrix $(\frac{\partial \alpha}{\partial x})$ exists. This is guaranteed by defining the transformation matrix $S(t)$ such that

$$(S^{-1})S = I \tag{24}$$

The point transformation matrix S(t) is given by

$$\Delta x = \begin{bmatrix} \frac{\partial x}{\partial \alpha} \\ \cdot \\ \frac{\partial \dot{x}}{\partial \alpha} \end{bmatrix} \Delta\alpha = S\,\Delta\alpha$$

$$S = \begin{bmatrix} -\frac{H}{v} & 0 & \frac{HXR}{h} & \frac{a^2 n}{h^2}\,HXR & -aR & -\frac{a}{r}\,R + \frac{\mu^2 \alpha_6 \alpha_4\,|a|}{h^2 r^2 v^2 na}\,HXR \\ 0 & \frac{H}{r} & \frac{H\dot{X}R}{h} & 0 & \frac{a}{2}\,\dot{R} & \frac{\mu a}{r^2 v^2}\,\dot{R} \end{bmatrix} \tag{25}$$

C. State Transition Matrix

The method of obtaining the state transition matrix is based on a generalization of an Encke method applied to linear prediction theory. It is assumed that the equations of motion may be decomposed into two factors

$$\dot{x} = g(x,t) + h(x,t)$$

where

$$g >> h .$$

It is further assumed that a closed form solution of the differential equations is known for the case where $h = 0$,

$$\dot{s} = g(s,t).$$

Furthermore, the state transition matrix for the approximating solution is known in closed form

$$\Delta s(t) = \phi(t,t_o)\ \Delta s(t_o) \qquad 26)$$

Let the deviation between the state variable and its approximation be given by

$$p(t) = x(t) - s(t) .$$

The perturbation equations of motion may now be written in the generalized Encke form

$$\dot{p} = g(x,t) - g(s,t) + h(x,t) \quad .$$

In order to guarantee that the deviation, p, is never permitted to grow too large, the process of rectification is introduced. Whenever a predetermined value of p is exceeded, the integration is terminated at time t_r. A new set of initial conditions are introduced, setting $p(t_r)$ equal to zero. Integration proceeds again about this new nominal approximate solution.

Since the deviation between x and s is never permitted to exceed the given value, the partial derivatives of the state variables from their nominal value may also be limited. Thus it is possible to write an approximate state transition matrix

$$\phi(t,t_o) \cong \psi(t,t_o) \qquad \text{for } t \leq t_r \quad . \tag{27}$$

Moreover, the approximate state transition matrix is known in closed form.

It may be necessary to perform a rectification during the period between observations. Following each such rectification, it is necessary to relate the state transition matrix at time t to the time of the previous observation. This may be accomplished by multiplying the approximate state transition matrix for times within each rectification interval by its value at the last rectification:

$$\psi(t,t_o) = \psi(t,t_r)\ \psi(t_r,t_o)$$

If rectification is performed in conjunction with a correction to the orbit through the minimum variance process, the matrix $\psi(t_r, t_o)$ is made the unit matrix.

D. <u>The Parameter Transition Matrix</u>

To obtain the transition matrix of the variational parameters, we note that

$$\frac{\partial\alpha(t)}{\partial\alpha(t_o)} = \sum \frac{\partial\alpha(t)}{\partial x(t)} \frac{\partial x(t)}{\partial x(t_o)} \frac{\partial x(t_o)}{\partial\alpha(t_o)} \tag{28}$$

In matrix form, this becomes

$$\Omega\,(t,\ t_o) = S^{-1}\,(t)\psi\,(t,\ t_o)\ S(t_o)\ .$$

In this manner, a form of the transition matrix may be obtained which does not violate the condition of energy dependence. The matrix, $\Omega\,(t,\ t_o)$ of the variational parameters is now given in closed form and only one element affects the energy.

The parameter transition matrix, $\Omega(t, t_r)$ is given below:

$$\Delta\alpha(t) = \Omega \cdot \Delta\alpha(t_r)$$

$$\begin{bmatrix} f\frac{v}{v_r} & -g\frac{v}{r_r} & 0 & 0 & 0 & 0 \\ -f_t\frac{r}{v_r} & g_t\frac{r}{r_r} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \ell_{34} & \frac{3}{2}\frac{\mu h a}{r^3 v^2}(t - t_r) & \ell_{36} \\ 0 & 0 & 0 & g_t & \frac{3}{2}\frac{a^2 n}{r}\ \alpha_6\ (t - t_r) & -\frac{r_r}{an} f_t \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\frac{an}{r} g & -\frac{3}{2}\frac{a^2 n}{r}\ \alpha_4\ (t - t_r) & \frac{r_r}{r} f \end{bmatrix} \qquad 30)$$

The terms ℓ_{34} and ℓ_{36} are given by

$$\ell_{34} = \frac{h}{v^2} \; a^2 n \left(\frac{g_t - 1}{r^2} - \frac{r_r f_t^{\,2}}{\mu} - \frac{f_t}{h^2} \left(\frac{\mu}{r} g - d_r \right) \right)$$

$$\ell_{36} = - \frac{h}{v^2} \; \frac{a}{r_r} f_t \left[\begin{array}{l} \dfrac{r_r}{r} f_4 \, (\theta) + \dfrac{r_r^{\,2}}{r^2} - \dfrac{\mu g_t}{r_r v_r^{\,2}} \\ \\ + \dfrac{\mu d_r}{h^2 r_r v_r^{\,2}} \, \alpha_6 \, (t_r) \, \left(\dfrac{\mu}{r} g - d_r \right) \end{array} \right] . \qquad 31)$$

To obtain the parameter transition matrix over a time interval greater than one rectification interval, the following equation applies:

$$\Omega(t, t_o) = \Omega(t, t_r) \, \Omega(t_r, t_o) \, . \qquad 32)$$

E. Partial Derivatives of the Observables

The program will accept the following types of observational data, singly or in combination:

1. Range
2. Range rate
3. Right ascension
4. Azimuth and elevation and Minitrack observations

In order to generate the differential corrections, it is necessary to compute residuals which consist of the difference between computed values of the observables and the observation data. In addition, it is necessary to compute partial derivatives of the observables with respect to the orbit parameters. The range, range rate, right ascension and declination can be expressed directly in terms of the geocentric state variables and the required partial derivatives may be obtained as follows:

Range: The computed value of the range is given by

$$\rho = \left[(x - x_s)^2 + (y - y_s)^2 + (z - z_s)^2\right]^{1/2} \tag{33}$$

and the matrix of partial derivatives for the range is given by

$$M(t) = \left[\frac{x - x_s}{\rho}, \frac{y - y_s}{\rho}, \frac{z - z_s}{\rho}, 0, 0, 0\right] \tag{34}$$

Range Rate: The computed range rate is given by

$$\dot{\rho} = \frac{1}{\rho}\left[(x - x_s)(\dot{x} + \omega_e y_s) + (y - y_s)(\dot{y} - \omega_e x_s) + (z - z_s)\dot{z}\right] \tag{35}$$

The matrix of partial derivatives of the range rate with respect to the state variables is given by

$$M(t) = \left[\frac{\dot{x} + \omega_e y_s}{\rho} - \dot{\rho}\,\frac{x - x_s}{\rho^2},\ \frac{\dot{y} - \omega_e x_s}{\rho} - \dot{\rho}\,\frac{y - y_s}{\rho^2},\right.$$

$$\left.\frac{\dot{z}}{\rho} - \dot{\rho}\,\frac{z - z_s}{\rho^2},\ \frac{x - x_s}{\rho},\ \frac{y - y_s}{\rho},\ \frac{z - z_s}{\rho}\right] \tag{36}$$

Right Ascension and Declination: The expressions for the right ascension and the declination may be written as

$$\sin D = \frac{z - z_s}{\rho}$$

$$\tan RA = \frac{y - y_s}{x - x_s} \tag{37}$$

The matrix of partial derivatives for D is given by

$$M(t) = -\frac{z - z_s}{\rho^2 \cos D}\left[\frac{x - x_s}{\rho},\ \frac{y - y_s}{\rho},\ \frac{(z - z_s)^2 - \rho^2}{\rho(z - z_s)},\ 0,\ 0,\ 0\right] \tag{38}$$

and for RA by

$$M(t) = \frac{1}{\sec^2 Ra}\left[-\frac{y - y_s}{(x - x_s)^2},\ \frac{1}{x - x_s},\ 0,\ 0,\ 0,\ 0\right] \tag{39}$$

Azimuth and elevation and the Minitrack observations are most conveniently expressed in a topocentric, local horizon coordinate system and, to treat them, it is useful to introduce the following relation between the topocentric and geocentric coordinates:

$$\begin{matrix} x''' \\ \\ y''' \\ \\ z''' \end{matrix} \quad = \quad \begin{bmatrix} \sin\varphi \cos\theta' & \sin\varphi \sin\theta' & -\cos\varphi \\ \\ -\sin\theta' & \cos\theta' & 0 \\ \\ \cos\varphi \cos\theta' & \cos\varphi \sin\theta' & \sin\varphi \end{bmatrix} \begin{matrix} x - x_s \\ \\ y - y_s \\ \\ z - z_s \end{matrix} \qquad 40)$$

This relation is used in developing the required partial derivatives for these angular observations.

Azimuth and Elevations: The expressions for azimuth and elevation are

$$A = \tan^{-1} \frac{y'''}{-x'''}$$

and 41)

$$E = \tan^{-1} \frac{z'''}{(\rho^2 - z'''^2)^{1/2}}$$

The corresponding matrices of partial derivatives are

For A

$$M(t) = - \frac{1}{\rho^2 - z''^{\prime 2}} [- x''' \sin\theta' - y''' \cos\theta' \sin\varphi ,$$

$$x''' \cos\theta' - y''' \sin\theta' \sin\varphi, y''' \cos\varphi, 0, 0, 0] \tag{42}$$

and for E

$$M(t) = \frac{1}{(\rho^2 - z''^{\prime 2})^{1/2}} \left[\cos\theta' \cos\varphi - \frac{z'''}{\rho^2} (x - x_s) ,\right.$$

$$\left. \sin\theta' \cos\varphi - \frac{z'''}{\rho^2} (y - y_s), \sin\varphi - \frac{z'''}{\rho^2} (z - z_s), 0, 0, 0\right] \tag{43}$$

Minitrack Observations: The Minitrack system direction cosines are expressed in terms of the topocentric coordinates as

$$\ell = - \frac{x'''}{\rho}$$

$$m = \frac{y'''}{\rho} \tag{44}$$

$$n = \frac{z'''}{\rho}$$

The corresponding matrices of partial derivatives are

for ℓ

$$M(t) = \frac{1}{\rho}\Big[- \sin\varphi \cos\theta' + \frac{x''{}'(x - x_s)}{\rho^2}, \; - \sin\varphi \sin\theta' + \frac{x''{}'(y - y_s)}{\rho^2},$$

$$\cos\varphi + \frac{x''{}'(z - z_s)}{\rho^2}, \; 0, \; 0, \; 0\Big] \qquad 45)$$

for m

$$M(t) = \frac{1}{\rho}\Big[- \sin\theta' - \frac{y''{}'(x - x_s)}{\rho^2}, \; \cos\theta' - \frac{y''{}'(y - y_s)}{\rho^2},$$

$$- \frac{y''{}'(z - z_s)}{\rho^2}, \; 0, \; 0, \; 0\Big] \qquad 46)$$

and for n

$$M(t) = \frac{1}{\rho}\Big[\cos\varphi \cos\theta' - \frac{z''{}'(x - x_s)}{\rho^2}, \; \cos\varphi \sin\theta' - \frac{z''{}'(y - y_s)}{\rho^2},$$

$$\sin\varphi - \frac{z''{}'(z - z_s)}{\rho^2}, \; 0, \; 0, \; 0\Big] \qquad 47)$$

The present program does not include onboard observations. It is recommended that in the extension for development of this program, this capability be added to the program.

VI. PROGRAM DESCRIPTION

The program has two independent operational modes. The first or reference mode concerns the generation of a reference trajectory and the simulation of radar observations associated with it. The second or determination mode determines a trajectory based upon observational data by the minimum variance filter. Since both modes involve the solution of the equations of motion by the modified Encke method, an explanation of this part of the program will precede a description of the two modes.

The equations of motion, as defined by equation 1, are given with respect to a reference body, the dominant central force field. The program assumes that this reference body is always the Earth. The program can integrate the Encke perturbation term for up to six orbits that have a common two-body solution. Although this is not done in the program, it can be included by defining separate initial conditions of the perturbation displacements for the six orbits. This feature allows for defining numerical approximations to analytical variation derivatives. The internal units that are used for all computations are earth radii and earth radii per hour.

A. Reference Mode

The purpose of this mode is to supply the state vectors and observation data that is used in the prediction mode. The observation data is output in the appropriate format on tape. An option is available to generate an ephemeris of the reference orbit on tape. The programming approach was to by-pass all computations that are not pertinent to this mode. When the final time is reached, the program returns for the next case.

Determination Mode

The minimum variance filter is used to determine the trajectory that corresponds to a set of observation data. An estimate of the initial conditions are input. The program continues integrating the equations of motion until the time of an observation. Based upon the current observations, the state variables are re-estimated. The program continues in this way, processing each observation as it occurs in time, until all the observations have been processed. It should be noted that the program is not presently capable of processing observation data at the initial time. When the data is exhausted, the reference mode is entered and the program continues in this mode until the final time is reached. When this occurs, a summary of the determination of the trajectory is given and the program then returns for the next case.

VII. OPERATING PROCEDURES

The program is loaded as a standard Monitor job on physical tape A-2 with the input following the program deck. Both the reference mode and the determination mode require the planetary tables tape on A-5. For the reference mode, the observations are output as binary information on L.T. 16. An option is available to generate an ephemeris of the reference trajectory on L.T. 18. When in the determination mode, observations may be supplied either on cards or on L.T. 16. Options are available to obtain a summary of the residuals in the observations and in the state variables. For the residuals in the observations a scratch tape is needed on L.T. 17. In order to obtain residuals in the state variables, the reference ephemeris is placed on L.T. 19 and a scratch tape on L.T. 18.

There is a program pause after the input has been read in for each case. Its purpose is to insure that the tapes have been mounted properly and also to give more flexibility to the user. This pause makes it possible to switch tapes from case to case. The pause condition is terminated by pressing the start key.

A. Treatment of the Planetary Coordinates

A tape of solar lunar and planetary coordinates based on Naval Observatory data is employed. It is in the form of overlapping two-year files, i.e., 1963-64, 1964-65, etc., with the coordinates of the various bodies referred to the mean equinox and equator of the middle of the two year file. The subroutine EPHEM searches the tape and reads in the proper file and record, keeping 30 days of tables in core storage at a time.

The first record on each file consists of the year in fixed decimal. Each of the following records contain the following information:

Word 1: Initial time of record in hours from base time.

Then 12 consecutive 15 word blocks containing the equatorial coordinates of:

XSE	YSE	ZSE	Sun wrt Earth
XJS	YJS	ZJS	Jupiter wrt Sun
XAS	YAS	ZAS	Mart wrt Sun
XVS	YVS	ZVS	Venus wrt Sun

Then three-30 word blocks containing:

XME	YME	ZME	Moon wrt Earth

The Moon coordinates are stored in half-day intervals (0.0^h, $12^h.0$ UT) unit of distance is the radius of the Earth. All other tables are in daily intervals (0^h. UT) the unit of distance being the AU.

At present, an ephemeris tape is available for 1961-1970, written in nine, two-year files. The files overlap one year.

The astronomical tables are stored in core in 24 hour intervals for the Sun and the planets and 12 hour intervals for the Moon. There are always 30 days of tables available arranged in such a way that the value of time for which the interpolation takes place is not near either end of the table. In Earth reference, the sequence of coordinates in the tables, all referred to the Earth as origin, is as follows:

In location "Table" there is the time of the first entry from the initial time. In "Table + 1" to "Table + 30" there are 30 x coordinates of the Sun.

In "Table + 31" to "Table + 60", the y coordinates of the Sun.

In "Table + 61" to "Table + 90", z coordinates of the Sun.

In "Table + 91" to "Table + 180", the x, y and z coordinates of Jupiter.

In "Table + 181" to "Table + 270", the x, y, z coordinates of Mars.

In "Table + 271" to "Table + 360", the x, y, z coordinates of Venus.

In "Table + 361" to "Table + 420", the x coordinates of the Moon.

In "Table + 421" to "Table + 480", the y coordinates of the Moon and in "Table + 481" to "Table + 540", z coordinates of the Moon.

VIII. INPUT

The data is entered in eleven sections. Each section is preceded by a heading card with the section number entered in cols. 1-5 as an integer. Each piece of input will be defined as one of the four categories, integer, fixed point, floating point or alphanumeric, by the notation, I, FX, FL or A. The quantity in the description column is entered on the specified card of the section in the appropriate columns. The name given is the name used for the quantity internally in the program.

Sect.	Card	Cols.	Name	Type	Description
1	1	2-72	ITITLE	A	Title
2	1	1-12	TIN	FL	Initial time, hr.
		13-24	TMAX	FL	Final time, hr.
		25-36	DTNE	FL	Integrating interval for near-earth portion of trajectory, hr.
		37-48	DTFE	FL	Integrating interval for far-earth portion of trajectory, hr.
		49-60	PRNTNE	FL	Print interval for near-earth, hr.
		61-72	PRNTFE	FL	Print interval for far-earth, hr.
	2	1-6	NYEARP	I	Year of trajectory
		7-12	DAYS	FX	Day of year
		13-18	HR	FX	Hr. of day
		19-24	HMIN	FX	Min. of hr.
		25-30	SEC	FX	Sec. of min.
	3	1-12	HMU	FL	Value of earth's gravitational constant, E.R.^{3}/hr.^{2}

Sect.	Card	Cols.	Name	Type	Description
		19-24	BMU	FX	6 values - represent planets - if planet is not used, insert 0 for that planet, if used insert 1. 1. Earth 2. Sun 3. Moon 4. Venus 5. Mars 6. Jupiter
3	1	1-5	MREF	I	Reference body (1-6)
		6-10	KLM	I	Indicator for dimension of input - R and $\dot{R}$ vectors may be input in 2 types of dimensions: 1. Earth radii, ER/hr. KLM = 0 2. km, km/sec. KLM = 1
		11-15	NUMSTA	I	No. of stations input
		16-20	KDATA	I	Number of time points at which you have observation data; if the observation data is input on cards, it is not necessary to have the exact count.
		21-25	KPRTR	I	Set to non-zero to generate a time history of the trajectory on L. T. # 18.
		26-30	IFLAG	I	Print control for complete Kalman calculation. (See output section for its use)
		31-35	IOBS	I	Indicator to compute observations. 0 = No observation 1 = Observation 2 = Observation and summary (uses tape # 17)

Sect.	Card	Cols.	Name	Type	Description
		36-40	LTBCD	I	BCD tape number for observation data.
		41-45	LTBIN	I	Binary tape number for observation data. 1. Computing observation of reference orbit = 16 2. Use Kalman scheme data on tape = 16 data on cards = 0
		46-50	KOND	I	Print control for additional trajectory information. (See output section for its use)
		51-55	INTPD	I	Indicator for Kalman scheme. INTPD = 2, process data all together at a time point. INTPD = 1, process each piece of data separately and pick best at each time point.
		56-60	NTEB	I	Number of ε^2 matrices
4	1	1-12	CONJR	FL	1st harmonic coefficient of the earth's potential.
		13-24	CONAR	FL	2nd harmonic coefficient of the earth's potential.
		25-36	CONKR	FL	3rd harmonic coefficient of the earth's potential.
		37-48	CDRAG	FL	Drag coefficient
		49-60	DCDRAG	FL	CDRAG increment for variation trajectory.
		61-72	AMASS	FL	Area/mass, cm^2/gm
5	1	1-12	VNAME	A	Name of vehicle
		13-24	RMIN	FL	Minimum perigee distance (E.R.)

Sect.	Card	Cols.	Name	Type	Description
		25-36	TADD	FL	A large number should be used to generate Gaussian noise on a clean data tape. Otherwise use zero.
		37-48	CLUE	FL	Inhibitor for Kalman scheme
		49-58	ITYPE	I	Type of observation
6	1	1-12	PSI60	FL	Greenwich hour angle of 1960, rad.
		13-24	PDOT	FL	Daily rate of earth's rotation, rad./day
		25-36	PSIDOT	FL	Hourly rate of earth's rotation, rad./hr.
		37-48	ERAD	FL	Equatorial radius of earth, km.
		49-60	EPSSQ	FL	Ellipticity of earth
		61-72	AUERAD	FL	Astronomical unit
7	1	1-36	RCIN	FL	x, y, z (dimension determined by KLM)
		37-72	RDCIN	FL	$\dot{x}$, $\dot{y}$, $\dot{z}$ (dimension determined by KLM)
8	1 per sta.	1-2	K	I	Station number
		3-14	STANM	A	Station name
		15-26	SLON	FX	Longitude, deg.
		27-29	SLONM	FX	Longitude, min.
		30-35	SLONS	FX	Longitude, sec.
		36-47	SLAT	FX	Geodetic latitude, deg.
		48-50	SLATM	FX	Geodetic latitude, min.
		51-56	SLATS	FX	Geodetic latitude, sec.
		57-68	SALT	FX	Geodetic altitude, ft.

Sect.	Card	Cols.	Name	Type	Description
9	1-6	1-72	PMAT	FL	(6 x 6 matrix) initial estimate of the covariance matrix. Each card has one row of matrix.
10	1	12 per value	TIMEB	FL	Array of times associated with various $\overline{\epsilon}^2$ matrices. Time is time from epoch in hrs.
	1-4	12 per value	TEBAR	FL	$\overline{\epsilon}^2$ matrix - 4 x 4 error variance matrix, as many matrices as times; each card a row of matrix. Angles in seconds, range in meters, and range rate in cm/sec.
11	0				The section card is used to end the input data for each run.

A. Observation Data

The observation data may be supplied either on cards or on tape. If the data is input on cards, a tape is generated with the appropriate format which may be used for subsequent runs. The formats for the tape and cards are described below. It should be noted that the order of the observations cannot be violated, i. e.,

1) Azimuth
2) Elevation
3) Range
4) Range rate
5) Right Ascension
6) Declination
7) open
8) open

9) open

10) open

The program is limited to ten types of observations.

B. Card Format

Since the format of observation cards vary with the source, the format itself is input on the first card. This is written exactly as a FORTRAN format statement except that the statement number and the word "Format" are eliminated. The program only requires that the information be ordered as follows:

1) Station number (as assigned in Section 8 of the input).
2) Time in days, hrs., mins., and seconds of the year.
3) Observations; blanks are used for missing observations.

   angles in degrees

   range in km.

   range rate in km./sec.

4) Type; see following.

C. Tape Format

A binary tape is supplied with the following information in each record.

1) Time from epoch in hrs., mins., and seconds.
2) Station number (as assigned in Section 8 of the input)
3) Type; see following.
4) Observations; blanks are not used for missing observations.

   angles in radians

   range in E. R.

   range rate in E. R./hr.

The observation type is specified by a sequence of ten values. Each value is associated with one of the ten types of observations in the reverse order of that given previously. A zero means the observation is <u>not</u> used and a one that it is used. The type may also be input in Section 5 of the input data. Its use here is to define the type to be generated when in the reference orbit mode. When in the prediction mode, a non-zero value is used to over-ride the type designation on the cards or tape.

IX. $\bar{\epsilon}^2$ MATRIX AND NOISE GENERATION

Since observation data quality may vary not only from station to station but also at a station, it is desirable to provide for such variations in function of time. Consider the input array TEBAR (10, 10, K) {K 10 x 10 matrices} where K associates the appropriate 10 x 10 matrix with a time in the following way: the minimum K such that the current time is less than TIMEB(K) is chosen. This particular 10 x 10 matrix represents the observation error variance matrix at that time. The diagonal terms are the error variances of azimuth, elevation, range, range rate, right ascension, declination and the rest are open. The off-diagonal terms are the covariances. This matrix is then stored in EBAR (I, J) in a packed manner. The terms involving observation types that are currently not of interest are deleted. For example, if the observation under consideration is one of range and range rate, EBAR is a 2 x 2 matrix; the first term on the diagonal is the variance of range, the second, the variance of range rate and the off-diagonal terms are their covariances.

When the observation data quality, EBAR, first changes it is multiplied by the input value CLUE, where CLUE is a large number. The method is useful in inhibiting the correction resulting from the minimum variance filter until more knowledge of the data quality is acquired. A gradual reduction of EBAR from observation to observation is performed as the information increases. EBAR is not allowed to go below the lower limit, the current TEBAR.

If the observations are error free, normally distributed random noise can be applied to the observations by inputing a non-zero value for

TADD. The mean of the distribution will be zero and its standard deviation is defined as the square root of the corresponding diagonal term of the current TEBAR. The subroutine ANDRN gives a description of the method used to generate the noise.

X. OUTPUT

The output is divided into three separate parts, each part being controlled by an input quantity that defines when that part should be output. The quantities, themselves, are all defined by name on the output but their dimensions are not. This will be mentioned below in the description of the three sections and how they are controlled by the associated input quantities. Obviously, if the program is operating in the reference mode the quantities that are not pertinent are deleted.

1. This section is controlledby the print intervals for near and far earth, PRNTNE and PRNTFE. These values determine the frequency with which this section is output. If the program is operating in the determination mode, this section will automatically be output at every observation time point. The quantities output are: The time, RC and RDC (vectors whose dimensions are determined by the input quantity KLM), and the ΔX correction vector to the state variables in E.R. and E.R./hr.

2. The frequency of this output is controlled by KOND in the following way:

   If KOND = -1 , this section is output every time section 1 is output. If KOND = 0, this section is never output. If KOND = N, then this section is output approximately every N minutes. The quantities output in this section are: the astronomical elements; observation information such as the station name, measured and computed values of the observables (angles in degrees, range in km, and range rate in km/sec); M matrix; $\bar{\epsilon}^2$ matrix; (angles in radians,range in earth radii, and range rate in earth radii per hour)

3. The frequency of this output is controlled by IFLAG in the same way that section 2 was controlled by KOND. The output in this section consists of the matrices involved in the minimum variance filter. They are output in the dimensions used internally in the machine. The matrices are S, $Q(t_{i-1})$, $\Omega(t,t_r)$, $\Omega(t,t_0)$, $Q(t_i)$, N, QN, Y, Y^{-1}, L, LNQ, and Q^+.

XI. GLOSSARY OF FORTRAN MNEMONICS

This is only a partial dictionary of the FORTRAN mnemonics and not in any sense intended to be complete. The FORTRAN terms that are frequently used or of particular interest are listed with the exception of the quantities that are input. The input quantities are described in the input section.

NAME	DESCRIPTION
A	a, semi-major axis
AABS	$\lvert a \rvert$
ALAMDA	parameter transition matrix, Ω, of the alphas
ALMAT	L matrix operator
AMMAT	M matrix, partial derivatives of the observations
BMU	indicators for the perturbing bodies
CA	$r\sqrt{\mu / \lvert a \rvert}$
CB	$\sqrt{\mu \lvert a \rvert}$
CC	$\sqrt{\mu / \lvert a \rvert}$
CD	$r / \lvert a \rvert$
CE	$R_o \cdot \dot{R}_o / \sqrt{\mu \lvert a \rvert}$
CF	$\frac{\lvert a \rvert}{\mu} R_o \cdot \dot{R}$
CKMER	conversion from E.R. to km
CKSERH	conversion from E.R./HR. to km/sec
CLUE	inhibitor for minimum variance
CWLIN	dimensioned block for integration routine containing functions, first and second derivatives
DAYK	number of full days from launch to observation time
DELALP	$\Delta \alpha$'s correction obtained from M. V. filter

DELX	Δ x 's corresponding to $\Delta \alpha$'s
DELY	observation values minus the computed values
DFRHO	linearization of $\frac{d\,\rho}{d\,h}$
DTI	actual integration interval
EN	n, mean motion
EBAR	$\bar{\epsilon}^2$ matrix
FA	f_1
FB	f_2
FC	f_3
FD	f_4
FPTH	$\frac{d\,F\,(\theta)}{d\,t} = F'\,(\theta)$
FRHO	density table
FTH	$F\,(\theta\,)$
HMU	μ
KWLIND	dimension of CWLIN array
KOND	rectification indicator
KDATA	number of records on the observational tape
KRUNT	number of simultaneous observations at any time
KSTA	station number corresponding to a particular set of observations
LTBIN	logical tape number of observation tape
MUD	error indicator
NOSOL	number of trajectories being integrated
NUMDAT	$\sum_{I=1}^{10}$ N TYPE (I)

PERDRG	perturbations due to drag
PEROBL	perturbations due to oblateness
PERRAD	perturbations due to radiation pressure
PMAT	P matrix
QMAT	Q matrix
RC	instantaneous position vector
RDC	instantaneous velocity vector
RCB	vectors from earth to perturbing bodies
RCIN	initial conditions on RC
RDCIN	initial conditions on RDC
RECTT	time of last rectification
RI	value of RC at last rectification
RDI	value of RDC at last rectification
RTAB	distance arguments table corresponding to density table
RTB	R two body
RDTB	$\dot{R}$ two body
RVB	vectors from vehicle to perturbing bodies
SDDXI	vectors of perturbations due to gravitation effects
SINMAT	S^{-1} matrix
SMAT	S matrix
STAC	geocentric station coordinate vector
TBF	f
TBG	g
TBFD	f_t

TBGD	g_t
TI	time of last rectification
TH	$\theta = E - E_o$
TK	time from epoch to the time of the observation in hours
YCOM	array of computed observations
YOBS	array of observed values

XII. SUBROUTINE DIRECTORY

NAME	DESCRIPTION
ANDRN	normally distributed random number generation
ATANS	computes arctangent in degrees
ATMSFR	prepares atmospheric density table
CROSS	computes cross product
CWLAR	used to keep the instantaneous, two body and perturbation vectors consistent
DERIV	computes the perturbation derivatives
DOT	computes dot product
DRAG	computes perturbation derivatives due to drag
EOFIX	avoids termination on EOF
EPHEM	Entry - (Planetary Position Routine) LAG: generates positions of the planets READ1: initial positioning of planetary tape REFSWT: re-initializes for a reference switch
FCOMP	computes f_1, f_2, f_3 and f_4
FIX	defines data type
INPUT	controls input
INT	controls entries to integration routine
KEPLER	computes two-body coordinates
MATINV	inverts a matrix
MDVECT	computes magnitudes of a vector
MINVAR	performs minimum variance matrix operations

OBLATE	computes perturbation derivatives due to oblateness
OBSER	computes observations
OSCUL	computes the osculating elements
PART	computes $\Omega (t, t_o)$
PRINT	controls output
RAPS	computes perturbation derivatives due to radiation pressure
RCTTST	tests for a rectification
RECORD	reads observation data
RECT	performs a rectification
REDUCE	reduces an angle to less than π
RPERG	computes magnitude of perigee vector
RWDE6F	Entry - (Integration routine)
	DECHA: sets up change in integration interval
	DEIN: initializes the integration
	DEREG: normal integration entry
	DERKI: Runge-Kutta integration
SMATRX	computes S matrix or its inverse
SUMARY	summarizes results of the run
VECTOR	computes magnitudes of a vector
WORKMU	prepares gravitational constants

Identification

ANDRN share Distribution AA-NDRN. A 704 SAP program that was Fortranized by John Mohan of AMA.

Purpose

To generate a sequence of normally distributed random numbers.

Method

Each entrance into ANDRN will yield one value. The value is obtained by first generating a psuedo-random number and then altering it to satisfy certain criteria that are explained in the share write-up. After many entrances to the routine, a sequence of numbers will have been generated that are characterized as normally distributed with the specified mean and standard deviation.

Usage

Calling sequence

ANS = ANDRN (σ, μ, x)

σ : standard devitation of the distribution

μ : statistical mean of the distribution

x : any large octal number. The number should be input to the main program and changed from run to run in order that unique sequences of pseudo-random numbers are generated from run to run.

Identification

ATANS - Fortran function

Purpose

Computes the arctangent of the argument with optional quadrant assignment.

Usage

ANS = ATANS (Y, X, K)

Computes arctangent of Y/X in degrees where Y and X have sign of sine and cosine respectively.

$K = 1 \quad 0 \leq \text{angle} \leq 360$

$K = 0 \quad -90 \leq \text{angle} \leq 90$

$K = -1 \quad -180 \leq \text{angle} \leq 180$

Identification

ATMSFR - Fortran subroutine

Purpose

This subroutine sets up an atmospheric model to be used when the inclusion of aerodynamic drag is desired. This is activated when the drag coefficient and the area-mass ratio of the vehicle are given as input. The atmospheric tables are stored in core. They correspond to model #7, contained in Report #25 (Reference 5) of the Smithsonian Astrophysical Observatory, fitted to the ARDC Model Atmosphere of 1956 (Reference 6) at low altitudes. The units for the air density are grams/cm^3 and the height is given in ER from the center of the earth.

Usage

CALL ATMSFR is performed in the initialization section of the main program.

Identification

CROSS - Fortran subroutine

Purpose

Compute $\overline{C} = \overline{A} \times \overline{B}$

where $\overline{A}$ and $\overline{B}$ are doubly subscripted.

Usage

CALL CROSS (A, B, C, J)

J: is second subscript on A and B.

Identification

CWLAR - Fortran subroutine

Purpose

This subroutine has two distinct functions:

1. computes $\xi = R_C - R_{TB}$

   $\dot{\xi} = \dot{R}_C - \dot{R}_{TB}$

2. computes $R_C = R_{TB} + \xi$

   $\dot{R}_C = \dot{R}_{TB} + \dot{\xi}$

Its principle use is to update the instantaneous position and velocity vectors after each integration.

Usage

CALL CWLAR

a (-1) in the list generates the first sequence of equations.

a (+1) in the list generates the second sequence of equations.

Identification DERIV

Computes the perturbation derivatives

Purpose

To compute all the contributions to $\ddot{\xi}$ and combine them, including the Encke term, the perturbations of the sun, moon, and planets, also the perturbation due to the non-spherical shape of the earth and the atmosphere.

Method

See equations 2 through 10.
See flow chart following description of this subroutine.

Symbols

CPOS block of reference body positions produced by LAG

RCB block of reference body positions as used by DERIV

RVB block of vehicle positions

CWLIN block containing ξ, $\dot{\xi}$ and $\ddot{\xi}$

SDDXI block of components of $\ddot{\xi}$ for each of the perturbing bodies and the Encke term

PEROBL oblateness components to $\ddot{\xi}$

PERDRG drag components

PERRAP radiation pressure (Dummy)

Note that XI, DXI, and DDXI do not appear with these names in the program. They are a part of the CWLIN array as specified by the integration subroutine. They may be obtained as follows:

XI(I, J) → CWLIN (1485 - 3•NØSØL + I + 3J)

DXI (I, J) → CWLIN (1485 - 6•NØSØL + I + 3J)

DDXI (I, J) → CWLIN (1485 - 9•NØSØL + I + 3J)

Arrangement of Storage Block For Integration Subroutine

RW DE6F	SPAT
DEQ	KWLIN
T	T
DELT	DTI
XI	CWLIN
DXI	
DDXI	
CWLIN	

XI(3,NØSØL)	→ CWLIN	(1488)
XI(1,1)	"	(1489-3·NØSØL)
DXI(3,NØSØL)	"	(1488-3·NØSØL)
DXI(1,1)	"	(1489-6·NØSØL)
DDXI(3,NØSØL)	"	(1488-6·NØSØL)
DDXI(1,1)	"	(1489-9·NØSØL)
CWLIN etc.	"	(1488-9·NØSØL)
	"	(1486-99.NØSØL)

The subroutine CWLAR (+1) gets the following:

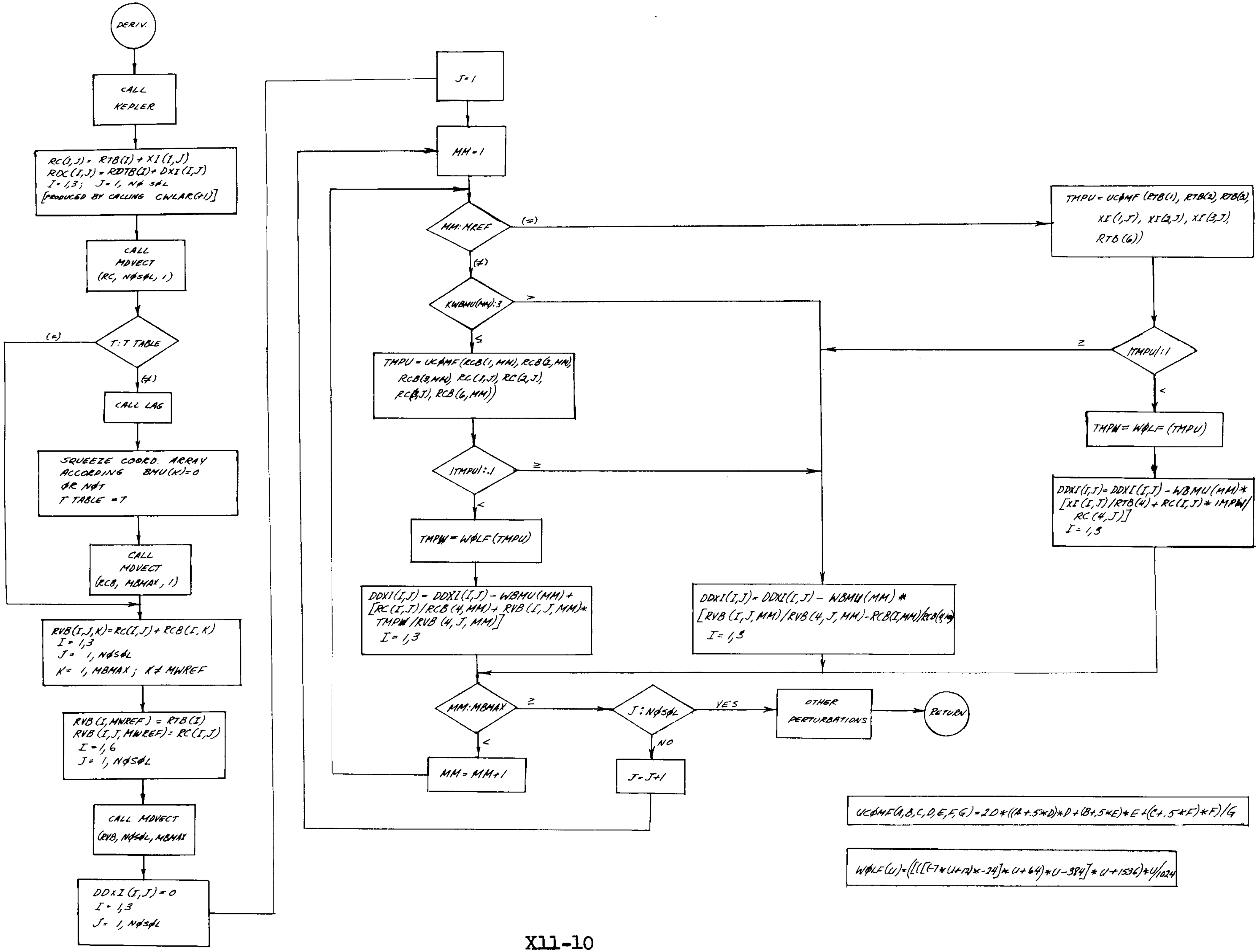
RC = RTB + XI

RDC = RDTB + DXI

while CWLAR (-1) gets:

XI = RC - RTB

DXI = RDC - RDTB


DERIV.
CALL KEPLER
RC(I,J) = RTB(I) + XI(I,J)
RDC(I,J) = RDTB(I) + DXI(I,J)
I = 1,3; J = 1, NØSØL
[PRODUCED BY CALLING CWLAR(+1)]
CALL MDVECT (RC, NØSØL, 1)
T:T TABLE
(=)
(≠)
CALL LAG
SQUEEZE COORD. ARRAY ACCORDING BMU(K)=0 ØR NØT
T TABLE = 7
CALL MDVECT (RCB, MBMAX, 1)
RVB(I,J,K)=RC(I,J)+RCB(I,K)
I = 1,3
J = 1, NØSØL
K = 1, MBMAX; K ≠ MWREF
RVB(I,MWREF) = RTB(I)
RVB(I,J,MWREF) = RC(I,J)
I = 1,6
J = 1, NØSØL
CALL MDVECT (RVB, NØSØL, MBMAX
DDXI(I,J) = 0
I = 1,3
J = 1, NØSØL
J = 1
MM = 1
MM:MREF
(=)
(≠)
KWBMU(MM):3
>
≤
TMPU = UCØMF(RCB(1,MM), RCB(2,MM), RCB(3,MM), RC(1,J), RC(2,J), RC(3,J), RCB(6,MM))
\|TMPU\|:.1
≥
<
TMPW = WØLF(TMPU)
DDXI(I,J) = DDXI(I,J) − WBMU(MM) + [RC(I,J)/RCB(4,MM) + RVB(I,J,MM)* TMPW/RVB(4,J,MM)]
I = 1,3
DDXI(I,J) = DDXI(I,J) − WBMU(MM) * [RVB(I,J,MM)/RVB(4,J,MM) − RCB(I,MM)/RCB(4,MM)
I = 1,3
TMPU = UCØMF (RTB(1), RTB(2), RTB(3), XI(1,J), XI(2,J), XI(3,J), RTB(6))
\|TMPU\|:1
≥
<
TMPW = WØLF (TMPU)
DDXI(I,J) = DDXI(I,J) − WBMU(MM)* [XI(I,J)/RTB(4) + RC(I,J) * TMPW/RC(4,J)]
I = 1,3
MM:MBMAX
≥
<
MM = MM+1
J:NØSØL
YES
NO
J = J+1
OTHER PERTURBATIONS
RETURN
UCØMF(A,B,C,D,E,F,G) = 2.0*((A+.5*D)*D+(B+.5*E)*E+(C+.5*F)*F)/G
WØLF(U) = ((((−7*(U+12)*−24)*U+64)*U−384)*U+1536)*U/1024

Identification

DOT - Fortran function

Purpose

This routine computes the dot product of two vectors that are doubly subscripted.

Usage

ANS = DOT (A, B, J)

where A and B are the two vectors and J is the second subscript of each vector.

Identification DRAG

Computes drag acceleration.

Purpose

To compute the perturbations contributed by atmospheric drag for as many as 15 trajectories, for one of which, designated JDRAG, the drag coefficient may be incremented.

Method

CALL DRAG

See equation 8. The density is obtained by linear interpolation of density-altitude table.

Symbols

VE	$\dot{R} - \Omega \times R$
FRHOA FRHOB	tables of $-\frac{1}{2}\rho\frac{A}{m} C_D$ versus altitude
DFRHOA DFRHOB	divided differences of the tables
POLAT	interpolated value of $-\frac{1}{2}\rho\frac{A}{m} C_D$ multiplied by $\lvert\dot{R} - \Omega \times R\rvert$
PERDAG	x, y, z components of drag acceleration

Identification

EOFIX FAP subroutine with two entries.

Purpose

This subroutine avoids termination by EXEM upon encountering an END OF FILE by adjusting EXEM.

Usage

```
CALL EOFIX(IND)
IF (IND) 1,2,1

1 ERROR RETURN;EOF ENCOUNTERED

2 READ TAPE NO.,LIST
```

CALL EXEMR to reset EXEM.

Identification

EPHEM	A FAP	subroutine with three entries.

1. READ1 — Ephemeris tape read and core set up routine
2. LAG — Interpolation for position and maintenance of current planetary positions.
3. RESULT — Reference ephemeris data to specified body.

Purpose

To set up in core a table of planetary positions referenced to a specifiable body, to read in additional data when necessary, to interpolate in the table for position, and to change the reference body when desirable.

Usage

1. READ1

Calling sequence (Fortran)

CALL READ1

The following common storage must be set up:

NYEAR : Year of launch

TZERO : Time in hours from base time*

DAYS : Launch time, days

HRS : Launch time, hours

MIN : Launch time, minutes

SEC : Launch time, seconds

MREF : Reference body

*Base time is 0.0 hours U.T. of December 31 of the year previous to launch.

Usage (continued)

LAG

Calling sequence (Fortran)

Call LAG

The following common storage must be set up

T - Time measured in hours from launch

READ1 must be called once to set tables in core. LAG will keep tables up to date.

REFSWT

Calling sequence (Fortran)

Call REFSWT

The following common storage must be set up

MREF - Reference body

READ1 gives position vectors with respect to the reference body indicated by MREF

MREF = 1	Earth reference
MREF = 2	Sun reference
MREF = 3	Moon reference
MREF = 4	Venus reference
MREF = 5	Mars reference

To change the reference body change MREF and CALL REFSWT

Identification

FCOMP - compute 6 functions of θ

Purpose

To compute f_1, f_2, f_3, f_4, F, and F' for use with the NASA two body equations.

Usage

1. Solution of Kepler's equation

The last of equation(12), which is a monotonically increasing function, is solved for θ by Newton's method as follows: letting θ_o = previous value of θ obtained, better values of θ are obtained by computing

$$\theta_{j+1} = \theta_j - \frac{F(\theta_j) - \Delta M}{F'(\theta_j)}$$

until convergence is attained. From θ and equation (12) plus equation (11) the position and velocity in the plane of the reference orbit are established.

2. The following series expansions for the f_1 and f_2 (equations (13) and (14)) are used:

$$\left.\begin{aligned} f_1 &= \theta - \sin\theta = -\theta\left\{\left(\left(\left(\frac{-\theta^2}{27\cdot 26}+1\right)\frac{-\theta^2}{25\cdot 24}+1\right)\ldots+1\right)\frac{-\theta^2}{3\cdot 2}\right\} \\ f_2 &= 1 - \cos\theta = -\left\{\left(\left(\left(\frac{-\theta^2}{26\cdot 25}+1\right)\frac{-\theta^2}{24\cdot 23}+1\right)\ldots+1\right)\frac{-\theta^2}{2\cdot 1}\right\} \end{aligned}\right] \text{Elliptic}$$

(A1)

$$\left.\begin{aligned} f_1 &= \sinh\theta - \theta = \theta\left\{\left(\left(\left(\frac{\theta^2}{27\cdot 26}+1\right)\frac{\theta^2}{25\cdot 24}+1\right)\ldots+1\right)\frac{\theta^2}{3\cdot 2}\right\} \\ f_2 &= \cosh\theta - 1 = \left(\left(\left(\frac{\theta^2}{26\cdot 25}+1\right)\frac{\theta^2}{24\cdot 23}+1\right)\ldots+1\right)\frac{\theta^2}{2\cdot 1} \end{aligned}\right] \text{Hyperbolic}$$

Elliptic Case

If $| \theta_c | < 1$

f_1 and f_2 are computed by Equations (A1)

$$\begin{aligned} f_3 &= \theta - f_1 \\ f_4 &= 1 = f_2 \end{aligned} \tag{A2}$$

If $| \theta_c | > 1$

Then f_4 is computed by means of Rand polynomials and

$$f_2 = 1 - f_4$$

Also if $| \theta - \sin \theta | < | \sin \theta |$

(or approximately if $| \theta | < 1.9$ (A3)

f_1 is computed by Equations (A1) and

$$f_3 = \theta - f_1$$

Otherwise if $| \theta - \sin \theta | > | \sin \theta |$

(or approximately if $| \theta | > 1.9$

f_3 is computed by means of Rand polynomials and

$$f_1 = \theta - f_3$$

Hyperbolic Case

If $|\theta| < 1.9$

f_1 and f_2 are computed by Equations (A1)

$$\begin{aligned} f_3 &= \theta + f_1 \\ f_4 &= 1 + f_2 \end{aligned} \tag{A4}$$

If $|\theta| > 1.$

Compute $f_o = e^{\theta}$

$$\begin{aligned} f_3 &= \tfrac{1}{2}\left(f_o - \frac{1}{f_o}\right) \\ f_4 &= \tfrac{1}{2}\left(f_o + \frac{1}{f_o}\right) \\ f_1 &= f_3 - \theta \\ f_2 &= f_4 - 1 \end{aligned} \tag{A5}$$

Identification

FIX FORTRAN subroutine

Purpose

To unpack observation data type designation.

Usage

Call FIX (OCT, N, NTYPE)

OCT: An octal number whose bits represent whether an observation type is available or not. A zero (0) means it is not; a one (1) that it is. Proceeding from right to left the bits represent azimuth, elevation, range, range rate, right ascension, and declination; the rest are open.

The program unpacks this number and stores each bit in the decrement of NTYPE which is dimensioned ten (10)

NTYPE (1) corresponds to azimuth

NTYPE (2) " " elevation

NTYPE (10) " " open

N: Represents the total number of observables available at any time point from one station.

Identification

INPUT a Fortran subroutine

Purpose

This routine controls the input to the program. The input is divided into eleven sections. When running consecutive cases only the section in which the input has changed need be input. This routine will also prepare the observation data tape if the observations are input on cards.

Usage

CALL INPUT

Identification

INT A Fortran subroutine.

Purpose

The routine was designed to make it easier to exchange the present integration package, RWDE6F , for another one. It has four distinct entries:

1. Initialization
2. Normal backward difference entry
3. Runge-Kutta entry
4. Change integration interval

Usage

The entries are achieved by using a 0, 1, 2, or 3 in the calling sequence. The subroutine RWDE6F should be read for a more detailed explanation of the linkage.

Identification

KEPLER Solution of the two body problem.

Purpose

To solve the NASA two body equations for a time, t, greater than or less than the time of rectification.

Method

1. Compute the increment in mean anomaly, $\Delta M = n(t-t_r)$

   If ,for the elliptic case, $|\Delta M| > 2\pi$, reduce ΔM by 2π and increase t_r by $2\pi/n$.

   Reduce θ to numerically less than π and adjust ΔM in the same way.

   For either the elliptic or the hyperbolic case, compute the functions f_1, f_2, f_3, f_4, F and F' (see description of FCOMP subroutine).

2. The equation

$$\theta_{K+1} = \theta_K - \frac{F - \Delta M}{F'}$$

   is iterated until

$$\frac{\theta_{K+1} - \theta_K}{\theta_{K+1}} < 4.E\text{-}07 \qquad \text{if } \theta_{K+1} > 1.E\text{-}04$$

   or until

$$\theta_{K+1} - \theta_K < 1.E\text{-}08 \qquad \text{if } \theta_{K+1} < 1.E\text{-}04$$

3. Compute f_1, ..., F' again; compute f, g, f_t, g_t and finally RTB and RDTB by use of equations (11) and (12).

Identification

MATINV Share distribution AN-F402, a 704 FORTRAN program.

Purpose

Solution of the inverse of a non-singular matrix.

Method

Gauss-Jordan elimination method is used to invert the matrix.

Usage

CALL MATINV (A,N,B,M,DETERM)

A: Matrix to be inverted
N: Order of the matrix A
B: Not defined for inverse solution but storage must be allocated.
M: A zero denotes that MATINV is to be used only for the inversion of the matrix A.

DETERM : determinant of the matrix A.

The inverse appears in A after return to calling program.

Identification

MDVECT a Fortran subroutine.

Purpose

This subroutine computes the 4th, 5th, and 6th terms of a two or three dimensional vector.

4th term: $|V|^3$

5th term: $|V|$

6th term: $|V|^2$

Usage

CALL MDVECT(V,I,J)

V : the vector of interest

I : the second subscript

J : the third subscript

Identification

MINVAR Fortran subroutine

Purpose

A variable order matrix multiplication routine specifically designed for the minimum variance matrix calculation. The variable order is included so that the observation data may be processed one at a time or all together. The subscript N in the description below is the quantity that may vary. It refers to the number of pieces of data that you wish to process through the minimum variance filter.

Usage

Enter with M_{Nx6}, S_{6x6}, $\bar{Q}_{6x6}$ and $\bar{\epsilon}^2_{NxN}$

Exit with Q^+_{6x6}, P^+_{6x6}, and L_{6xN}

Subroutine computes

$$N_{Nx6} = M_{Nx6} \; S_{6x6}$$

$$A_{6xN} = Q^-_{6x6} \; N^*_{6xN}$$

$$Y_{NxN} = N_{Nx6} \; A_{6xN} + \bar{\epsilon}^2_{NxN}$$

$$L_{6xN} = A_{6xN} \; Y^{-1}_{NxN}$$

$$Q^{+}_{6x6} = Q^{-}_{6x6} - L_{6xN} \; N_{Nx6} \; Q^{-}_{6x6}$$

$$P^{+}_{6x6} = S_{6x6} \; Q^{+}_{6x6} \; S^{*}_{6x6}$$

Identification

OBLATE a Fortran subroutine

Purpose

To compute the perturbations contributed by the non-spherical shape of the earth.

Method

See equations (6) and (7).

The earth oblateness potential may be written as

$$\phi = \frac{-\mu}{r} \left\{ \frac{J_{20}}{r^2} \left[\frac{3}{2} \left(\frac{z}{r}\right)^2 - \frac{1}{2} \right] + \frac{J_{30}}{r^3} \left[\frac{5}{2} \left(\frac{z}{r}\right)^3 - \frac{3}{2} \left(\frac{z}{r}\right) \right] \right.$$

$$\left. + \frac{J_{40}}{r^4} \left[\frac{35}{8} \left(\frac{z}{r}\right)^4 - \frac{15}{4} \left(\frac{z}{r}\right)^2 + \frac{3}{8} \right] \right\}$$

where the equatorial radius of the earth is the unit of length and where the coefficients J_{20}, J_{30}, J_{40} are assigned the following numerical values (given by equations (8) and (13) of reference 7):

$$J_{20} = 1082.3 \times 10^{-6}$$

$$J_{30} = -2.3 \times 10^{-6}$$

$$J_{40} = 1.8 \times 10^{-6}$$

The perturbation accelerations due to the earth's oblateness are given by equations (6) and (7) of this document. As actually programmed in the minimum variance program, the functions b and c of equation (7) appear as

$$b = \frac{CONJ}{r^5}\left[\frac{5}{2}\left(\frac{z}{r}\right)^2 - \frac{1}{2}\right] + \frac{CONA}{r^6}\left[\frac{15}{2}\left(\frac{z}{r}\right) - \frac{35}{2}\left(\frac{z}{r}\right)^3\right]$$

$$+ \frac{CONK}{r^7}\left[-63\left(\frac{z}{r}\right)^4 + 42\left(\frac{z}{r}\right)^2 - 3\right]$$

$$c = \frac{CONJ}{r^5}\left[-z\right] + \frac{CONA}{r^5}\left[\frac{15}{2}\left(\frac{z}{r}\right)^2 - \frac{3}{2}\right] + \frac{CONK}{r^6}\left[-12\left(\frac{z}{r}\right) + 28\left(\frac{z}{r}\right)^3\right]$$

where

CONJ	=	$3\mu J_{20}$	=	$2\mu J_2$	of equation 7
CONA	=	$-\mu J_{30}$	=	μJ_3	of equation 7
CONK	=	$-\frac{5}{8}\mu J_{40}$	=	$5\mu J_4$	of equation 7

Note that the corresponding input quantities to the program are

CONJR	=	$\frac{3}{2}\mu J_{20}$	=	μJ_2	of equation 7
CONAR	=	$-\mu J_{30}$	=	μJ_3	of equation 7
CONKR	=	$-\frac{15}{4}\mu J_{40}$	=	$30\mu J_4$	of equation 7

Identification

OBSER a Fortran subroutine

Purpose

This routine computes the observables for a particular station and the residuals which consist of the differences between the computed values of the observables and the observation data. It also computes the partial derivatives of the observables with respect to the state variables. It is capable of this for azimuth, elevation, range, range rate, right ascension and declination in any combination. If the program is operating in the data generating (reference) mode it will generate a binary tape containing the observations, the time of the observation, the type of observations made and the station from which the observations were made. This tape may then be used as a simulation of real data for the orbit determination mode of the program. If the program is operating in the determination mode it will generate another tape that contains the observations, the residuals and the data type so that a summary of this information may be made at the completion of the case.

Usage

CALL OBSER (K, NTYPE)

K : The station number that was assigned the station in the input.

NTYPE : A specification of the combination of data types. This is explained in subroutine FIX.

Method

See equations (33) through (47).

Identification

OSCUL Fortran Subroutine

Purpose

The purpose of this routine is to compute the classical osculating Keplerian elements.

Usage

CALL OSCUL (T, RC, RDC, HMU, J, MUD, SCALED)

- T: time
- RC: position vector, doubly subscripted
- RDC: velocity vector, doubly subscripted
- HMU: gravitational constant
- J: refers to the second subscript on RD, RDC which defines the particular variation trajectory of interest
- MUD: error return
- SCALED: output of lengths will be scaled by this quality.

Equations

The osculating elements are obtained from the following equations:

$$a = \left(\frac{2}{r} - \frac{v^2}{\mu} \right)^{-1}$$

$$n = \mu^{\frac{1}{2}} a^{-3/2}$$

$$\left. \begin{array}{l} e \cos E \\ e \cosh E \end{array} \right\} = 1 - \frac{r}{a}$$

$$\left. \begin{array}{l} e \sin E \\ e \sinh E \end{array} \right\} = \frac{d}{\sqrt{|\mu a|}}$$

$$M = \left\{ \begin{array}{l} E - e\sin E \\ e \sinh E - E \end{array} \right.$$

$$t_{PC} = t - \frac{M}{n}$$

The angles Ω, ω, i are obtained from the vectors H and $\hat{P}$, where

$$H = R \times \dot{R}$$

$$e\hat{P} = \left(\frac{1}{r} - \frac{1}{a} \right) R - \frac{d}{\mu} \dot{R}$$

In terms of these vectors:

$$\cos i = \frac{H_z}{h} \qquad \text{i in the first or fourth quadrant}$$

$$\sin \Omega = \frac{H_x}{h \sin i}$$

$$\cos \Omega = \frac{-H_y}{h \sin i}$$

$$\cos \omega = P_x \cos \Omega + P_y \sin \Omega$$

$$\sin \omega = \frac{P_z}{\sin i}$$

Identification

PART Fortran Subroutine

Purpose

This subroutine computes the parameter transition matrix, Ω, at any time. It must be called at rectification. If the rectification is due to a minimum variance correction, it sets $\Omega = I$. See discussion following equation (27).

Usage

CALL PART

The parameter transition matrix, Ω, is given by equation (30).

Identification

PRINT a FORTRAN subroutine

Purpose

This subroutine controls the output concerning the trajectory of the vehicle. It will output R_{VC} and R_{VC} in E.R. and E.R./hr or KM and KM/hr depending upon the input option KLM. The time at which output is given is determined by PRNTDT, a print frequency that is a function of the input parameters PRNTNE and PRNTFE. This routine also controls the output of the classical astronomical elements. The frequency of this output is explained in the Output section.

Usage

CALL PRINT

Identification

RAPS a FORTRAN subroutine

Purpose

A dummy subroutine that is intended to compute the perturbation accelerations due to solar radiation pressure.

Usage

CALL RAPS

Identification

RCTTST A Fortran subroutine.

Purpose

The two-body orbit is rectified whenever the perturbations exceed specified maximum values. The following three tests are made:

1. $$\frac{|\xi|}{|R_C|} > .002$$

2. $$\frac{|\dot{\xi}|}{|R_C|} > .002$$

3. $$u = \frac{2}{r_{TB}^2}\left[R_{TB} + \frac{\xi}{2}\right] \cdot \xi > .05$$

Usage

CALL RCTTST

The value KOMP, which is in COMMON, will contain an integer which corresponds to the test that was failed upon return to calling program. Otherwise KOMP = 0 .

Identification

RECORD a Fortran subroutine

Purpose

The reading of the observation data tape is performed by this routine. It is read so that there are always two records in core at any one time. A record consists of a time, observation data and the associated station and the observation data type. If there is more than one record at the same time due to simultaneous observations from different stations, the routine will read ahead to see how many records have the same time. It will return to the main program with this count in COMMON. After an observation is read noise will be applied as defined in subroutine ANDRN if the option has been taken. The observations and the associated $\overline{\epsilon^2}$ matrix are packed in this routine to facilitate processing through the minimum variance filter.

Usage

CALL RECORD (NTYPE)

NTYPE : A specification of the combination of data types. This is explained in subroutine FIX.

Identification

RECT a FORTRAN subroutine

Purpose

This subroutine computes the parameters pertinent to a rectification. The three basic terms that are computed are a, n and $R_r \cdot \dot{R}_r$ where the subscript r refers to the value of R_c and $\dot{R}_c$ at the time of rectification. It also computes various combinations of these values that are defined in the Glossary of Fortran Names.

Usage

CALL RECT

Identification

REDUCE   Fortran function

Purpose

Reduces an angle to less than π

Usage

```
ANGLE  =  REDUCE (ANGLE)
```

Identification

RPERG a FORTRAN function

Purpose

Computes the magnitude of the perigee vector.

Usage

ANS = RPERG (J,HMU)

J: Refers to the particular variation orbit of interest.

HMU: The gravitational constant associated with the particular reference body from which the perigee distance is being computed.

Equation

Perigee distance = a(1-e)

a: Semi-major axis

e: eccentricity

Identification

RWDE6F — Share Distribution No. 775. A 704 SAP program that was Fortranized by Leon Lefton of AMA.

Purpose

To solve a set of N simultaneous second order differential equations.

Method

A fourth-order Runge-Kutta method is used to start the integration and to change the step-size during integration. A Cowell "second-sum" method based on sixth order differences is used to continue the integration. Double precision is used internally to control round-off errors. Truncation error can be controlled by choosing an appropriate step-size, or by using the variable step-size mode of operation. The values of the variables and derivatives are consistent at all times.

Usage

1) For initialization

CALL DEIN (NEQ, DERIV, IORD, K, EPS, HMIN, HMAX, YMIN, DPT, ACC)

- NEQ : Number of differential equations
- DERIV: Name of derivative routine
- IORD: Order of backward difference scheme
- K : Ratio of Cowell step size to Runge-Kutta step size
- EPS : Convergence criterion
- HMIN : Minimum step size

HMAX : Maximum step size

YMIN : Minimum function value allowed

DPT : Least significant part of time

ACC : Routine places + value when in Runge-Kutta and - value when in Cowell.

2) Normal entry

CALL DEREG

3) Change integration interval

CALL DECHA (Nt)

4) Runge-Kutta integration with specified Δt, CALL DERKI

The location of NEQ is considered as the address that begins the block of information concerning RWDE6F. Following NEQ there is

time

Δt

$y_1 - N$

$\dot{y}_1 - N$

$\ddot{y}_1 - N$

The share write up should be read for a detailed description of RWDE6F.

Identification

SMATRX Fortran subroutine

Purpose

This subroutine computes the point transformation matrix S or its inverse.

Usage

CALL SMATRX

A +1 in the list will generate S

A -1 in the list will generate S^{-1}

The S matrix and its inverse are given in equations (23) and (25).

Identification

SUMARY a Fortran subroutine

Purpose

This routine summarizes the residuals in observations and/or the state variables. The residuals in the observations are output next to the observations themselves. The time, in hours, min., and seconds, from epoch is also given. The residuals in the state variables may also be obtained if a tape of the reference orbit ephemeris from which the observations were generated is supplied on L.T-19. The time, as defined above, is given. The component errors, magnitude error and also the relative error is given for the position and velocity vectors. The relative error is the magnitude of the error vector divided by the magnitude of the vector. All results are labeled by name and dimension.

Usage

CALL SUMARY (I,J,K)

I : Number of observations to be summarized

J : Set equal to two (2) to obtain observation residuals

K : Set equal to one (1) to obtain state variable residuals.

Identification

VECTOR a Fortran subroutine

Purpose

The routine computes the 4th, 5th and 6th terms of a one dimensional vector.

$$4\text{th term} = |V|^3$$

$$5\text{th term} = |V|$$

$$6\text{th term} = |V|^2$$

Usage

CALL VECTOR (V)

V is the vector of interest.

Identification

WORKMU Fortran subroutine

Purpose

The program is capable of including the gravitational attraction of six perturbing bodies.

1. Earth
2. Sun
3. Moon
4. Venus
5. Mars
6. Jupiter

If any of these bodies are not desired, (see input section #2, card #3), this subroutine will pack the various arrays that are associated with these bodies to simplify the logic in generating the gravitational perturbations.

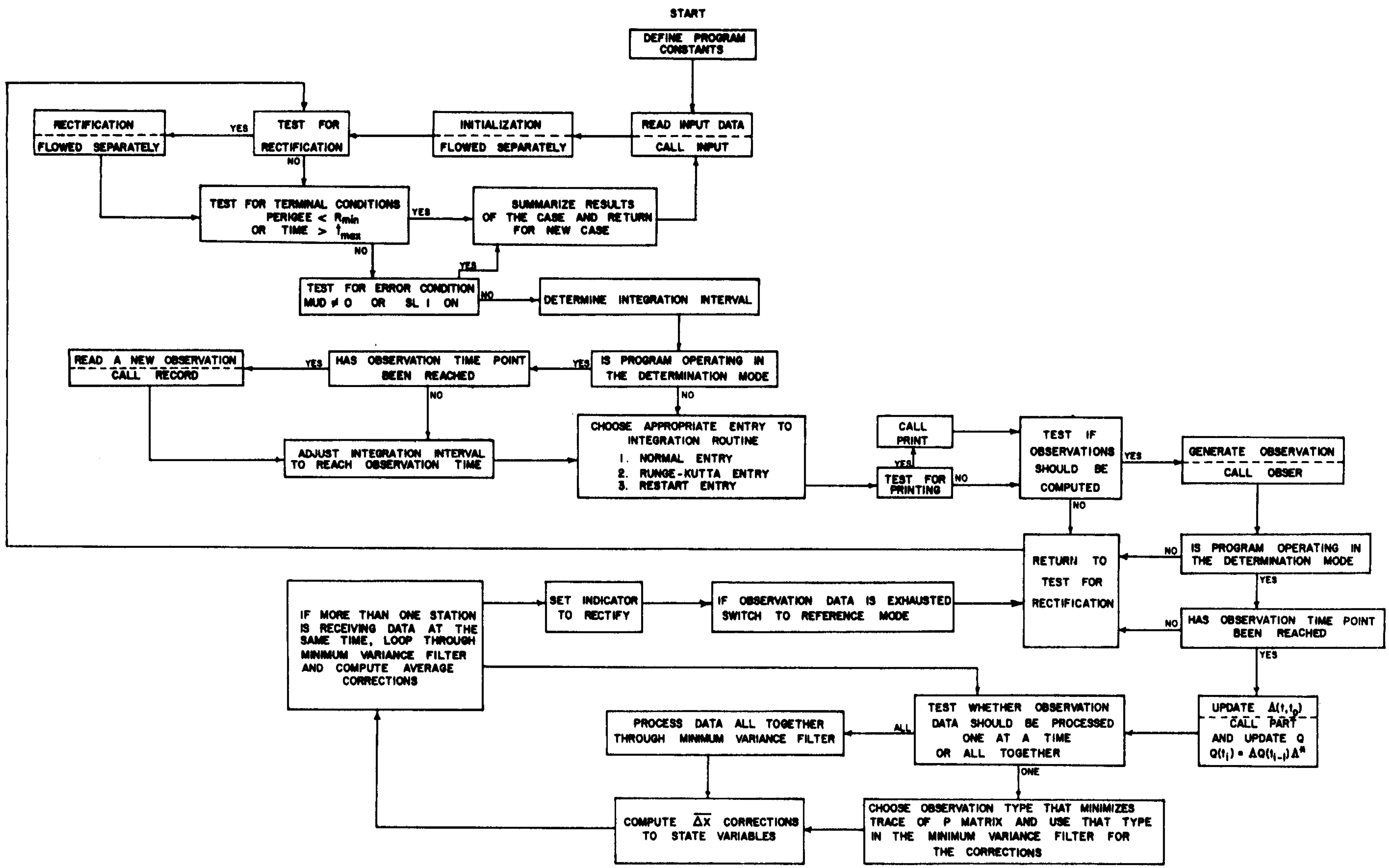
Usage

CALL WORKMU is performed in the initialization section of the main program.

Symbols

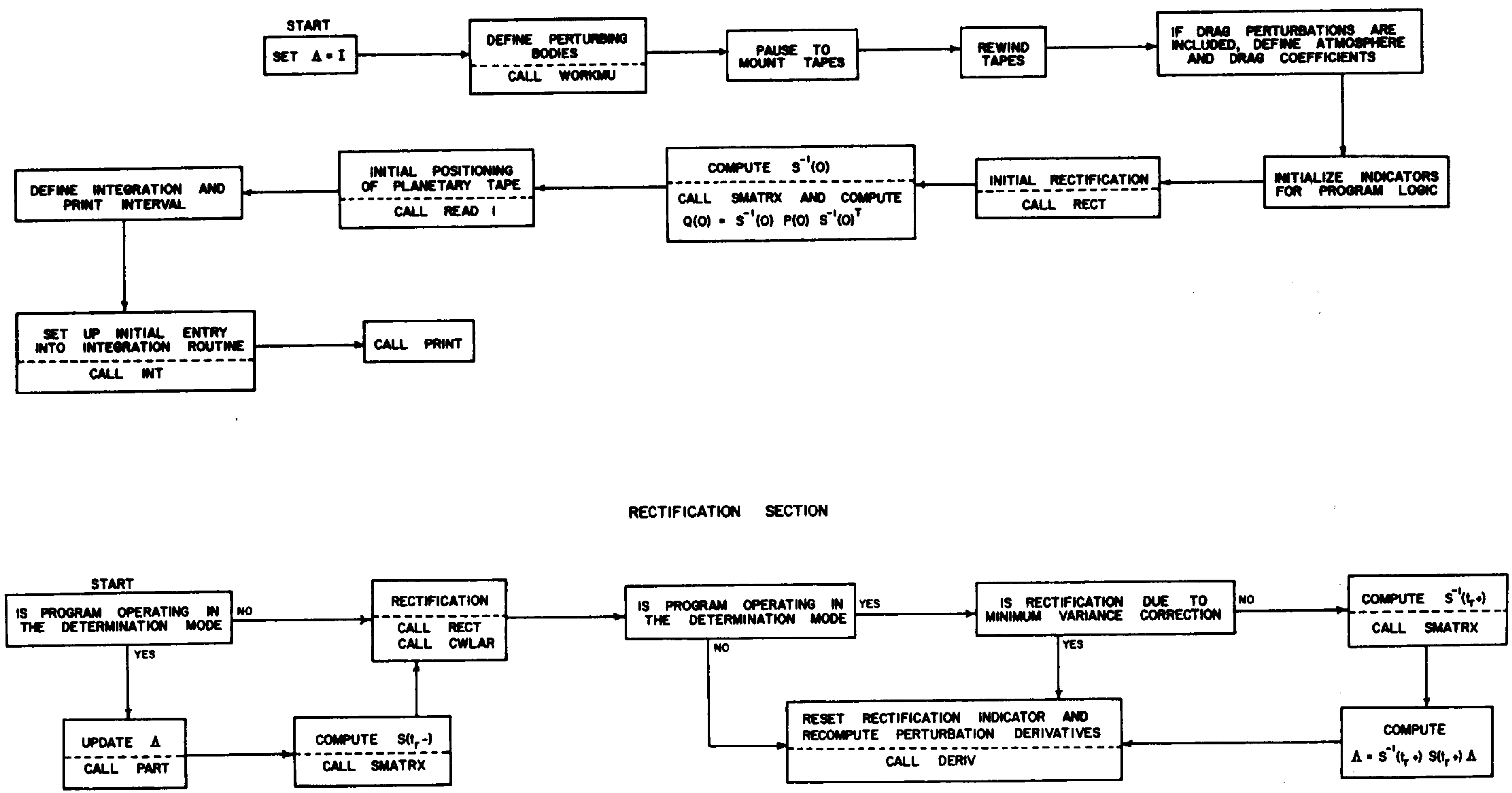
HMU	μ of reference body
BMU	table of 6 mass parameters
WMBU	table of working mass parameters
BNAME	names of 6 objects
MBMAX	number of working objects
KWBMU	table relating indexing of working objects to the original 6

MAIN PROGRAM



MAIN PROGRAM

INITIALIZATION SECTION



RECTIFICATION SECTION

REFERENCES

1. Minimum Variance Precision Tracking and Orbit Prediction Program, Final Report. Contract No. NAS 5-2535. Prepared by Analytical Mechanics Associates, Inc. for Goddard Space Flight Center. May 1963.

2. Asymptotically Singular Differential Correction Matrices. S. Pines, H. Wolf, M. Payne. NASA NAS 5-293, Republic Aviation Corporation.

3. Goddard Minimum Variance Orbit Determination Program. Report No. X-640-62-191, Special Projects Branch, Theoretical Division, October 18, 1962.

4. Interplanetary Trajectory Encke Method, Program Manual, Report No. X-640-63-71, May 1, 1963, Goddard Space Flight Center, Greenbelt, Maryland.

5. The Structure of the High Atmosphere, C. A. Whitney, Smithsonian Institution, Astrophysical Observatory, Special Report, No. 25, April 20, 1959.

6. The ARDC Model Atmosphere, 1956, R. A. Minzner, W. S. Ripley, December, 1956. AFCRCTW-56-204, Astia Doc. 110233.

7. A Review of Geodetic Parameters, W. M. Kaula, NASA Report X-640-63-55, May, 1963, Goddard Space Flight Center, Greenbelt, Maryland.